Maximal persistent current in a type-II superconductor with an artificial pinning array at the matching magnetic field

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The current carrying steady state of the pinned flux line lattice created by magnetic field is described. We calculate analytically the critical current for the case of the matching field (when the number of vortices is equal to that of the pinning centers) using a simple variational method in the framework of Ginzburg-Landau equations. The vortex cores are deformed and displaced in the current carrying state. Displacement of the centers of the vortices with respect to pinning centers and structure of these states are determined.

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I. INTRODUCTION

The great interest in the problem of magnetic flux pinning in type-II superconductors is associated with its relevance to technological applications of superconductivity. An important challenge in applications of type-II superconductors is achieving optimal critical currents under given magnetic fields. This requires preventing depinning of Abrikosov vortices during formation of the resistive state under the applied current. Random pointlike pinning centers naturally appear due to imperfections of lattice structure or chemical disorder. However, in technologically important materials critical current due to intrinsic pinning is not strong enough, especially at high magnetic fields. One of the main reasons is destructive competition of pinning centers, as demonstrated by the collective pinning theory.[1] It was predicted theoretically[2] and confirmed experimentally[3–5] that when pinning centers are arranged into a periodic array commensurate with the Abrikosov lattice, the critical current increases dramatically. The effect is maximized when the filling fraction f (defined as a ratio between number of vortices to that of the pinning centers) is 1, when one pinning center traps a single vortex. Additional vortices are "interstitial" and can be depinned easily, thus significantly reducing the critical current.[6] Recently there has been an advance in the fabrication method of the periodic arrays of pinning sites.[7] The arrays with triangular, square, and rectangular geometries have been fabricated using either microholes or blind holes,[3] magnetic dots,[4] and columnar defects.[5]

Theoretically these systems were studied, using mostly numerical methods, within a model of interacting two-dimensional (2D) points representing vortices subject to pinning potential.[8,9] This approach is appropriate to describe weak magnetic fields and sparse pinning arrays, so that the structure of the vortex core can be ignored. Recently, however, the arrays are fabricated on the nanometer scale, and the range of fields applied continuously increases. Therefore the distribution of the order parameter becomes of importance and one has to resort to a more fundamental approach. Since microscopic approach is not practical, the only available tool is the Ginzburg-Landau (GL) phenomenological approach.[1] Within this approach the periodic pinning problem was tackled numerically by Priour and Fertig.[10] They demonstrated that the pinning centers deform the vortex core and, moreover, that the current carrying state for a large square-shaped pinning center displaces a vortex in the direction perpendicular to the persistent current. Unfortunately, only one vortex and one pinning center were simulated on the square sample with area carrying just one unit of flux $\Phi_0=\frac{hc}{2e}$, while the rest of the vortex-pinning center pairs were represented by periodic boundary conditions duplicating the "squares." It is well known that in an isotropic superconductor the intervortex repulsion (which is rather strong at elevated fields) forces them to form a hexagonal vortex lattice.[1] The square lattice will therefore be in conflict with these forces and the question is whether this is an important factor in the pinning problem. In addition, it is clear that in order to maximize the critical current the pinning center array should be hexagonal with one pinning center per vortex (the matching field) and this is the situation we consider in the present analytical calculation.

In this paper we employ the GL equations for the order parameter Ψ in order to determine the persistent current and to describe the structure of the pinned vortex matter in superconducting films at matching field ($f=1$). The sample is considered to be infinite in directions perpendicular to applied magnetic field, so that the vortex-vortex interactions are fully accounted for. Current carrying states of the flux lattice at matching field and hexagonal array of pinning centers are characterized by displacement of the vortices with respect to that of the pins and by deformations of the vortex cores. A variational method exploiting the two lowest Landau levels (LLLs) is used to determine the displacement and to calculate the critical current at the matching field for rectangular pinning centers of arbitrary aspect ratios. The dependence of the persistent current on the displacement is linear at small displacements and approaches its maximum (the critical current) at about half of the intervortex distance. The coefficient in the linear part, the Labusch constant, is calculated.

II. GINZBURG-LANDAU EQUATIONS WITH A PERIODIC PINNING ARRAY

Let us consider a type-II superconducting film of width s under constant magnetic field $\mathbf{H}$ perpendicular to the film.

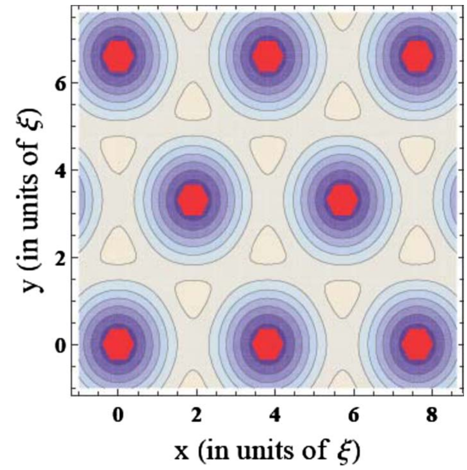

FIG. 1. (Color online) A 2D array of pinning centers commensurate with hexagonal Abrikosov lattice (superfluid density is shown) at matching field. All the currents are diamagnetic, no persistent current present.

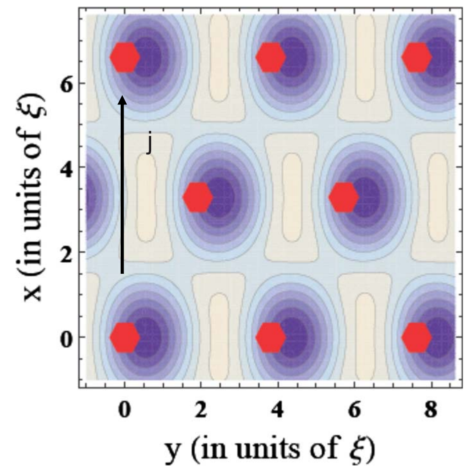

FIG. 2. (Color online) A persistent current carrying state. Vortices are displaced with respect to the pinning centers. The current density **J**, in addition to the diamagnetic component, has a relatively small persistent current component. A variational order parameter configuration includes two lowest Landau levels.

Static magnetic properties of the superconductor are described by the GL Gibbs energy[1] as a function of the order parameter Ψ and vector potential $\mathbf{A}$,

$$F[\Psi,\mathbf{A}] = s\int d\mathbf{r}\left[\frac{\hbar^2}{2m^*}|\mathbf{D}\Psi|^2 - a'(\mathbf{r})|\Psi|^2 + \frac{b'}{2}|\Psi|^4 + \frac{1}{8\pi}(\mathbf{B}-\mathbf{H})^2\right]. \tag{1}$$

Here $\mathbf{D}\equiv\nabla+i(2\pi/\Phi_0)\mathbf{A}$ denotes the covariant derivative, where $\Phi_0=hc/e^*$, $e^*=2|e|$ is the unit of flux, $\mathbf{B}=\nabla\times\mathbf{A}$ is the magnetic induction, and m^* is the effective mass. Assuming that the ratio $\kappa_{eff}\equiv\lambda_{eff}/\xi\gg 1$, where $\lambda_{eff}=2\lambda^2/s$ is the effective penetration depth and ξ is the coherence length, magnetization is by a factor $1/\kappa^2$ smaller than the field. Consequently for magnetic fields few times larger than H_{c1}, one uses $B\approx H$. The vector potentials are chosen in the symmetric gauge,

$$A_x = -\tfrac{1}{2}By, \quad A_y = \tfrac{1}{2}Bx. \tag{2}$$

The pinning centers are located at points $\mathbf{r}_a$ [2D vectors $\mathbf{r}=(x,y)$ will be denoted by bold letters] (see Fig. 1). When pinning is absent, the coefficient $a'(\mathbf{r})=\alpha(T_c-T)$ in Eq. (1) is uniform. The static free energy is minimized by a hexagonal Abrikosov lattice of vortices with cores, with primitive vectors of hexagonal lattice

$$\mathbf{a}_1 = a_\Delta\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right), \quad \mathbf{a}_2 = a_\Delta(1,0), \tag{3}$$

where the lattice spacing is $a_\Delta=2^{1/2}3^{-1/4}\sqrt{\Phi_0/B}$. Pinning is represented by an inhomogeneous coefficient

$$a'(\mathbf{r}) = \alpha(T_c-T) - T_c\sum_a U(\mathbf{r}-\mathbf{r}_a), \tag{4}$$

where U are "potentials" around pinning centers $\mathbf{r}_a$. As discussed above, an interesting configuration corresponds to a hexagonal periodic array located at

$$\mathbf{r}_a = n_1\mathbf{a}_1 + n_2\mathbf{a}_2 \tag{5}$$

commensurate with the static Abrikosov lattice at matching field.

The superconducting current density has a form

$$\mathbf{J} = \frac{ie^*\hbar}{2m^*}(\Psi^*\mathbf{D}\Psi - \Psi\mathbf{D}\Psi^*). \tag{6}$$

In the pinned state considered in the present paper electric field is absent outside of very narrow shelf near the boundaries[1] (of the width of order of ξ). Therefore there is no normal current present in the bulk of the sample, so that $\mathbf{J}$ represents both the persistent and the diamagnetic (namely, the one circling around the vortex cores) components of the supercurrent density. The persistent current, which originates from the normal electron's current in the leads, is seen as an imbalance between the currents on two sides of vortices (see Fig. 2). We will be mostly interested in the sample average of the current density [assumed to be along the y direction (see Fig. 1)],

$$J = \frac{1}{L_xL_y}\int d^2\mathbf{r}J_y(\mathbf{r}) \equiv \langle J_y\rangle. \tag{7}$$

Only the persistent current component contributes to it.

Below the coherence length $\xi=\hbar/(2m^*\alpha T_c)^{1/2}$ will be used as a unit of length $\mathbf{r}\to\mathbf{r}/\xi$ and $H_{c2}=\Phi_0/2\pi\xi^2$ as a unit of magnetic field, $h=B/H_{c2}$. The scaled order parameter $\psi=2^{-1/2}\Psi/\Psi_0$, where $|\Psi_0|=(\alpha T_c/b')^{1/2}$, so that the dimensionless energy can be written in the following form:

$$f_{\rm GL} = \int d^2\mathbf{r}(f_2+f_4+f_p), \tag{8}$$

$$f_2 = \psi^*\left[-\frac{D^2}{2} - \frac{1-t}{2}\right]\psi, \tag{9}$$

$$f_4 = \frac{1}{2}(\psi^*\psi)^2, \qquad (10)$$

$$f_p = \sum_a V(\mathbf{r}-\mathbf{r}_a)\psi^*\psi, \qquad (11)$$

where $t=T/T_c$ and $V(\mathbf{r})=U(\mathbf{r}/\xi)/\alpha$ is dimensionless pinning potential. It is convenient to present the persistent current in the units of the depairing current $J_d=cH_{c2}/2\pi\xi\kappa^2$.The dimensionless of the current density is defined by

$$\mathbf{J} = J_d\mathbf{j} \qquad (12)$$

with

$$\mathbf{j} = \frac{i}{2}[\psi^*\mathbf{D}\psi - \psi(\mathbf{D}\psi)^*]. \qquad (13)$$

Below the energy Eq. (8) is minimized with fixed average current [Eq. (7)] first by using a simple variational procedure.

III. VARIATIONAL METHOD FOR PERSISTENT CURRENT CARRYING PINNED STATES

A. Qualitative description and symmetry considerations

Neglecting the vortex creep due to thermal fluctuations on the mesoscopic scale, the vortex matter in the presence of pinning can be in one of the two stationary states, either a pinned (static) vortex lattice or a flux flow. Both states generally carry the electric current; however the nature of conductivity is totally different. In the pinned state the transport is dissipatedness due to the "persistent" superconducting current and electric field vanishes inside the sample. When the persistent current in the pinned state approaches a critical magnitude J_c, vortices are depinned and the flux flow ensues. In the flux flow regime the electric field does penetrate the bulk of the superconductor and Ohmic (Bardeen-Stephen) dissipation arises.

In the absence of persistent current at the matching field the Abrikosov lattice coincides ("matches") with the pinning array. In particular vortex centers (zeros of the order parameter) will coincide with the pinning centers at $\mathbf{r}_a$ (see Fig. 1). Persistent current not only displaces the "vortices" with respect to the pinning centers (by the Lorentz force) but also significantly deforms their shape[10] (see Fig. 2). At small persistent current densities the displacement of vortices should be small enough, so that linear elasticity theory applies. For increasing current densities nonlinear effects appear and grow. Eventually at a critical current density the static distorted Abrikosov lattice becomes unstable to depinning. The critical current and the order parameter configuration at given current density depend both on the strength and on the shape of the pinning center.

Free energy of a pinned system in which there is one vortex per pinning site ($f=1$) [Eq. (8)] has a hexagonal lattice translational symmetry

$$\mathbf{r} \to \mathbf{r} + n_1\mathbf{a}_1 + n_2\mathbf{a}_2, \qquad (14)$$

where the lattice vectors were defined in Eq. (3). In addition it has a rotation by $\pi/6$ symmetry. In the presence of the persistent current (see Fig. 2), the configuration of the order parameter in principle might not be symmetric (namely, a spontaneous symmetry breaking takes place). However numerical simulations we made demonstrate that in the present case spontaneous symmetry breaking of the translational symmetry [Eq. (14)] does not occur. This greatly constrains possible solutions and allows a simple variational procedure. The rotational hexagonal symmetry is broken spontaneously down to reflection symmetry with respect to the supercurrent direction.

B. Why a lowest Landau level order parameter configuration cannot carry net supercurrent in a pinned state

Let us consider a simple case of short-range pinning potential and then generalize to arbitrary shape of the pinning center. It is clear that with such a choice of pinning potential a properly normalized LLL state,

$$\psi(\mathbf{r}) = \sqrt{\frac{a_h}{\beta_A}}\varphi_0(\mathbf{r}),$$

$$\varphi_0(\mathbf{r}) = 3^{1/8}\sqrt{h}\sum_i e^{i\pi l^2/2}\exp\left\{i\left[-\frac{h}{2}xy + \frac{\pi(2l+1)}{a}\left(x-\frac{a}{4}\right)\right] - \frac{h}{2}\left[y-\frac{\pi(2l+1)}{ah}\right]^2\right\}, \qquad (15)$$

where $a=a_\Delta/\xi$, $a_h=(1-t-h)/2$, is still an approximate Abrikosov solution of the GL equation for zero net supercurrent. It seems natural to try to look for periodic configurations of the order parameter describing the current carrying states among the other LLL states. It is well known that an LLL is uniquely defined by locations of its zeros, so that possible candidates are displaced configurations, $\varphi_0(\mathbf{r}+\mathbf{u})$. It turns out that this naive assumption fails since generally these states have vanishing persistent current. Indeed it can be shown[8] that in any LLL state the current density is proportional to curl of superfluid density

$$J_i \propto \varepsilon_{ij}\partial_j|\psi|^2, \qquad (16)$$

so that an integral over unit cell vanishes due to periodicity. Here ε_{ij} is an antisymmetric tensor in two dimensions. Consequently to describe states carrying the persistent supercurrent, higher Landau levels are necessary.

C. Trial functions

The simplest such states involve, in addition to the LLL, just a shifted first Landau level,

$$\psi = \sqrt{\frac{a_h}{\beta_A}}[c_0\varphi_0(\mathbf{r}+\mathbf{u}) + ic_1\varphi_1(\mathbf{r}+\mathbf{u})], \qquad (17)$$

where

$$\varphi_1(\mathbf{r}) = 2^{1/2}3^{1/8}h\sum_l \left[y - \frac{\pi(2l+1)}{ah}\right]e^{i\pi l^2/2}\exp\left\{i\left[-\frac{h}{2}xy + \frac{\pi(2l+1)}{a}\left(x-\frac{a}{4}\right)\right] - \frac{h}{2}\left[y - \frac{\pi(2l+1)}{ah}\right]^2\right\}. \quad (18)$$

Functions φ_N are normalized by $\langle|\varphi_0(\mathbf{r})|^2\rangle = 1$. Variational coefficients c_0 and c_1 are assumed to be real for the following reasons. First the overall phase does not influence gauge invariant quantities and second the supercurrent is directed along the y axis only when the relative phase is $\pi/2$. Substituting Eq. (17) into the current density defined in Eq. (13) and using

$$D_y\varphi_0 = \left(\frac{\partial}{\partial y} + i\frac{h}{2}x\right)\varphi_0 = -\sqrt{\frac{h}{2}}\varphi_1, \quad (19)$$

one obtains

$$\langle j_y\rangle = \frac{a_h}{\beta_A}\sqrt{2h}c_1c_0. \quad (20)$$

This means that the persistent current appears due to a mixture of LLL and the first LL (see Fig. 2). Similarly when higher LLs are present the current will get contributions due to mixture of φ_N and φ_{N+1} only ["a selection rule" (see Ref. 8 for description)]. Therefore higher Landau levels $N>1$ will contribute very small corrections because, as mentioned above, the Abrikosov LLL solution is dominant. We therefore should minimize the free energy [Eq. (8)] for a set of variational parameters c_0, c_1, and $\mathbf{u}$.

IV. STRUCTURE OF THE PINNED STATE AND CRITICAL CURRENT FOR VARIOUS PINNING CENTERS

A. Minimization of energy

The variational energy (averaged over the unit cell) has the quadratic in order parameter, quartic and the pinning contributions defined in Eq. (8),

$$\langle f_2\rangle = \frac{a_h}{\beta_A}[(h-a_h)c_1^2 - a_hc_0^2], \quad (21)$$

$$\langle f_4\rangle = \frac{a_h^2}{2\beta_A^2}\langle[c_0^2|\varphi_0|^2 + c_1^2|\varphi_1|^2 + ic_0c_1(\varphi_0^*\varphi_1 - \varphi_0\varphi_1^*)]^2\rangle = \frac{a_h}{\beta_A}\left[\frac{a_hc_0^4}{2} + \frac{a_h\beta_1}{2\beta_A}c_1^4 + 2\frac{a_h\beta_{12}}{\beta_A}c_0^2c_1^2\right], \quad (22)$$

$$\langle f_p\rangle = \frac{a_h}{\beta_A}[c_0^2\rho_0 + c_1^2\rho_1 + 2c_0c_1\rho_{12}], \quad (23)$$

where $\langle\cdots\rangle$ is an average over sample (equivalently over the unit cell due to translational symmetry discussed above). The last contribution involves integrals over the pinning potential (obviously assuming nonoverlapping pinning potentials),

$$\rho_0 = \langle V(\mathbf{r})|\varphi_0(\mathbf{u}+\mathbf{r})|^2\rangle; \quad \rho_1 = \langle V(\mathbf{r})|\varphi_1(\mathbf{u}+\mathbf{r})|^2\rangle, \quad (24)$$

$$\rho_{12} = \frac{i}{2}\langle V(\mathbf{r})\varphi_0^*(\mathbf{u}+\mathbf{r})\varphi_1(\mathbf{u}+\mathbf{r})\rangle + \text{c.c.} \quad (25)$$

The constants in Eq. (22) are

$$\beta_1 = \int dxdy|\varphi_1|^4 = 2.1126,$$

$$\beta_{12} = \int dxdy|\varphi_0|^2|\varphi_1|^2 = \beta_A/2 = 0.58. \quad (26)$$

The minimization equations with respect to variational parameters of the trial functions c_0 and c_1 are

$$\frac{df_{\text{GL}}}{dc_0} \propto a_h(-1 + c_0^2 + c_1^2)c_0 + c_0\rho_0 + c_1\rho_{12} = 0, \quad (27)$$

$$\frac{df_{\text{GL}}}{dc_1} \propto \left[(h-a_h) + \frac{a_h\beta_1}{\beta_A}c_1^2 + a_hc_0^2\right]c_1 + c_1\rho_1 + c_0\rho_{12} = 0. \quad (28)$$

The displacement of vortices $\mathbf{u}$ determines the transport current, so that to find the critical current density, one has to maximize the current when $\mathbf{u}$ runs over the unit cell (see Fig. 3). This set of equations is solved numerically in two cases: the δ potential and rectangular pins. In particular it is instructive to solve the equations using perturbation theory in small displacement $\mathbf{u}$ (linear elasticity theory). Since the current will be flowing along the y direction, the displacement vector should be oriented in the x direction.

B. Location of vortices for the δ-pinning array: Linear elasticity theory

For simplicity let us consider first an array of identical pinning centers each of which is described by the potential

$$V(\mathbf{r}) = U\sum_a \delta(\mathbf{r}-\mathbf{r}_a). \quad (29)$$

This is appropriate when the size (radius) of the pin w is smaller than the coherence length ξ. The dimensionless pinning strength parameter can be estimated by $U = \pi w^2\varepsilon/T_c$, where $\varepsilon \sim T_c - T_{c0}$ is potential well energy. In this case integrals in Eqs. (24) and (25) are simply

$$\rho_0 = U|\varphi_0(-\mathbf{u})|^2, \quad \rho_1 = U|\varphi_1(-\mathbf{u})|^2,$$

$$\rho_{12} = \frac{i}{2}U[\varphi_0^*(-\mathbf{u})\varphi_1(-\mathbf{u}) - \varphi_0(-\mathbf{u})\varphi_1^*(-\mathbf{u})]. \quad (30)$$

We start from derivation of the linear elasticity of the pinned vortex lattice in which the displacement (which is along the x direction) is assumed to be small.

There is only one stable solution for $u_x = u = 0$: $c_0 = 1, c_1 = 0$ (there is another unstable solution with $c_0 = 0$). This is just the Abrikosov lattice configuration with zero critical current (higher order corrections in a_h involve 6th, 12th, etc., Landau levels, which are very small and well beyond our variational procedure). Expanding functions (15)

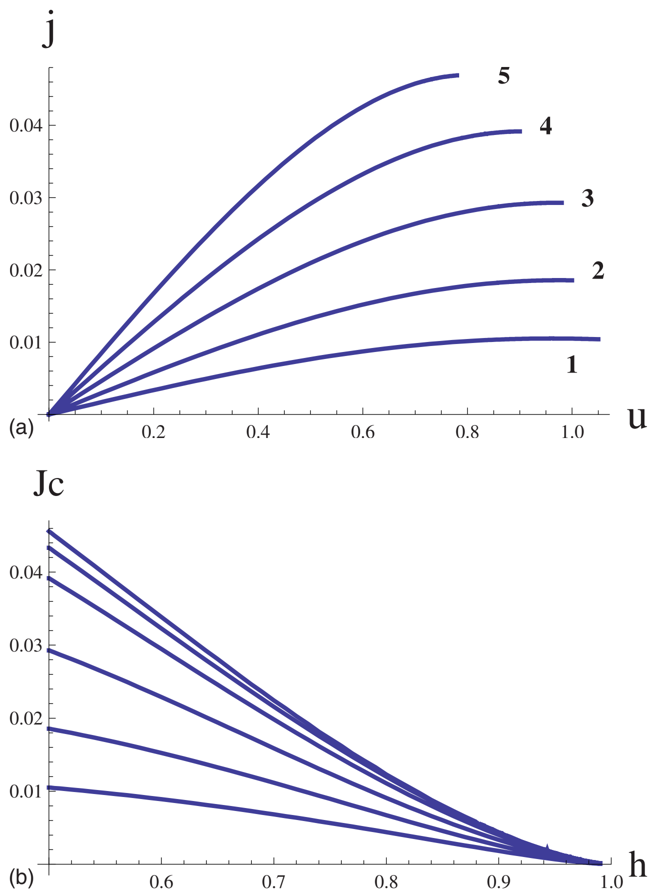

FIG. 3. (Color online) (a) Dependence of the dimensionless persistent current on displacement for the delta pinning for different pinning strengths (curve 1 $U=0.05$, curve 2 $U=0.1$, curve 3 $U=0.2$, curve 4 $U=0.4$, and curve 5 $U=1$). (b) Dependence of the maximal persistent current on the magnetic field for the delta pinning for different values of pinning strengths $U=0.05, 0.1, 0.2, 0.4, 0.6, 0.8$ (from bottom to top).

and (18) in small displacement u around this equilibrium solution one obtains for densities Eq. (30) appearing in the pinning energy,

$$\rho_0 = U|\varphi_0(-u)|^2 \simeq \frac{U}{2h}|\varphi_1(0)|^2 u^2 + O(u^3),$$

$$\rho_1 \simeq U|\varphi_1(0)|^2 + O(u),$$

$$\rho_{12} = \frac{iU}{2}[\varphi_0^*(-u)\varphi_1(-u) - \varphi_0(-u)\varphi_1^*(-u)]$$

$$= -\frac{U}{2(2h)^{1/2}}|\varphi_1(0)|^2 u + O(u^2), \tag{31}$$

where $|\varphi_1(0)|^2=3.77h$.

To the leading nontrivial order therefore one obtains $c_0=1+\varepsilon_0 u^2$, $c_1=\varepsilon_1 u$. Substituting this into the minimization equation [Eq. (27)], one obtains to order u^2

$$a_h(2\varepsilon_0 u^2 + \varepsilon_1^2 u^2) + \rho_0 + \varepsilon_1 u \rho_{12} = 0. \tag{32}$$

Using Eq. (31), one obtains

$$a_h(2\varepsilon_0 + \varepsilon_1^2) + \frac{U|\varphi_1(0)|^2}{2h} - \varepsilon_1 \frac{U|\varphi_1(0)|^2}{2\sqrt{2h}} = 0. \tag{33}$$

Similarly the second minimization equation [Eq. (28)] to the first order in u reads

$$h\varepsilon_1 + U|\varphi_1(0)|^2\varepsilon_1 - \frac{1}{2}\sqrt{\frac{1}{2h}}U|\varphi_1(0)|^2 = 0, \tag{34}$$

determining the correction

$$\varepsilon_1 = \frac{1}{2}\sqrt{\frac{1}{2h}}\frac{U|\varphi_1(0)|^2}{U|\varphi_1(0)|^2 + h}. \tag{35}$$

Consequently the average persistent current density responsible for the pinning force is

$$j = \frac{a_h}{\beta_A}\sqrt{2h}c_1 = \frac{a_h}{2\beta_A}\frac{U|\varphi_1(0)|^2}{U|\varphi_1(0)|^2 + h}u. \tag{36}$$

The Lorentz force on one vortex depends linearly on displacement $j=Ku$, where the Labusch parameter is

$$K = \frac{a_h}{2\beta_A}\frac{3.77U}{3.77U+1}. \tag{37}$$

This parameter enters various phenomenologically important quantities such as surface impedance of the microwave absorption.[11]

C. Beyond the elasticity theory: Critical current

Let us now turn to the calculation of the critical current. It is natural to assume that above certain current density the static solution loses its stability. The linear elasticity, which breaks down well below this critical current density, is reached (within a harmonic well depinning is actually impossible and formally the critical current is infinite). To estimate the critical current, the minimization equations therefore should be solved numerically. In Fig. 3(a) we show dependence of the current on the displacement of vortices in the x direction for different values of pinning strength U for $h=0.5$ and $t=0$ (that is, for $a_h=0.25$). For small displacement u the persistent current density rises linearly consistently with perturbation theory. In addition it is clear that the current vanishes when vortex stays right in the middle between the pinning centers, that is, for $u=a_\Delta/2$. Therefore the maximal persistent current j_c should exist at certain displacement u_c in between. The displacement u_c weakly depends on pinning strength decreasing as the pinning strength rises. In the critical current one observes that at $U<0.1$ the critical current is well approximated by

$$j_c = \frac{a_h}{\beta_A}U. \tag{38}$$

For stronger pinning the vortex is hold tightly (confined) by the pining center and j_c diverges as in the linear elasticity theory. For pinning centers of the order or larger than coherent length ξ the model should be generalized. In addition the shape of large pinning center might become important.

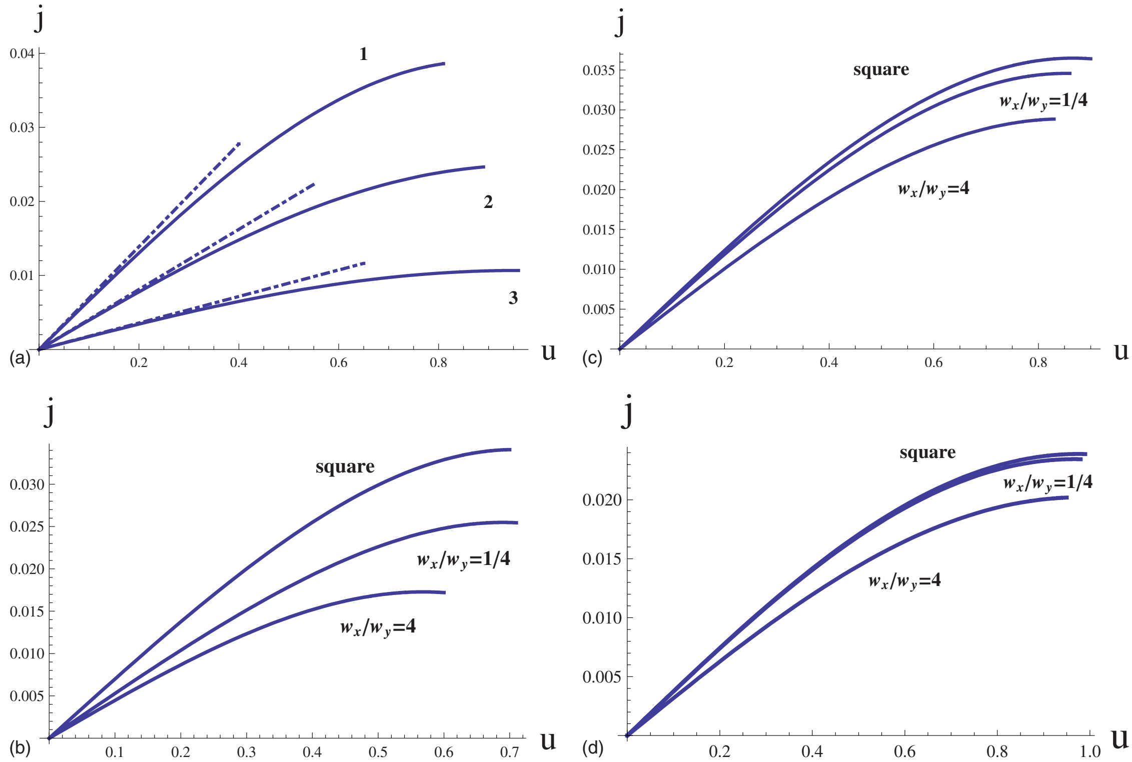

FIG. 4. (Color online) (a) Dependence of the persistent current on displacement for the rectangular pinning centers for small area of the pinning center and three different pinning strengths (curve 1 $U=3$, curve2 $U=1$, and curve 3 $U=0.33$). The straight lines correspond to elasticity theory [Eq. (37)]. (b) Dependence of the persistent current on displacement for various shapes of pinning centers for fixed pinning potential $V_0=3$ and area $w_x w_y=0.5$. (c) Dependence of the persistent current on displacement for various shapes of pinning centers for fixed pinning potential $V_0=1$ and area $w_x w_y=0.5$. (d) Dependence of the persistent current on displacement for various shapes of pinning centers for fixed pinning potential $V_0=0.33$ and area $w_x w_y=0.5$.

D. Rectangular pinning center array

A commensurate array of rectangular artificial pinning centers can be modeled using the following single pin potential:

$$V(\mathbf{r}) = \begin{cases} V_0 & \text{for } -w_x/2 < x < w_x/2 \text{ and } -w_y/2 < y < w_y/2 \\ 0 & \text{otherwise.} \end{cases} \quad (39)$$

In this case the integrals appearing in the pining term [Eqs. (24) and (25)],

$$\rho_0 = \int_{-w_x/2}^{w_x/2} dx \int_{-w_y/2}^{w_y/2} dy |\varphi_0(\mathbf{u}+\mathbf{r})|^2,$$

$$\rho_1 = \int_{-w_x/2}^{w_x/2} dx \int_{-w_y/2}^{w_y/2} dy |\varphi_1(\mathbf{u}+\mathbf{r})|^2,$$

$$\rho_{12} = \frac{i}{2}\int_{-w_x/2}^{w_x/2} dx \int_{-w_y/2}^{w_y/2} dy \varphi_0^*(\mathbf{u}+\mathbf{r})\varphi_1(\mathbf{u}+\mathbf{r}) + \text{c.c.}, \quad (40)$$

were performed numerically. The dependence of the current on displacement for different aspect ratios $r=w_x/w_y$, pinning areas $S=w_x w_y$, and the potential strength U is given in Fig. 4. Magnetic field and temperature are fixed as before at $h=0.5$, $t=0$, so that $a_h=0.25$. The results are discussed below.

V. DISCUSSION AND CONCLUSIONS

The model of the vortex crystal pinned by the periodic array of inclusions at the matching magnetic field considered here is remarkably simple and useful for theoretical analysis. It allows us to study, from first principles, various general properties of the vortex matter including critical current, elasticity, and destruction of the vortex crystal. Dependence of the dimensionless persistent current on the displacement of the vortex lattice with respect to pins for the δ pinning was calculated analytically [see Fig. 3(a)]. In order to express the results in physical units the dimensionless current density must be multiplied by the depairing current density $J_d=cH_{c2}/2\pi\xi\kappa^2$. At small pinning strength U the critical current increases rapidly and saturates at large U [see Fig. 3(b)]. The shape of the pinning center also affects the critical current when both the size of the pinning center (on the scale of coherence length) and its strength are sufficiently large (see Fig. 4). The critical current is always largest for a more symmetric square pin [lines numbered 1 on Figs. 4(a) and 4(b)]. It is followed by a rectangular center with the long side parallel to the current [lines numbered 2 on Figs. 4(a) and 4(b)]. The rectangular pinning centers with the long side perpendicular to the current [lines numbered 3 on Fig. 4(a)] have always the lowest critical current. At small pinning potential the difference is insignificant, as can be learnt from Figs. 4(c) and 4(d). The results for the smaller square pinning centers are consistent with the δ-function approximation, as can be seen from the linear part of the dependence of the current on displacement in Figs. 4(b) and 4(d). The lines follow a simple formula for the Labusch parameter [Eq. (38)]. The dependence of the largest persistent current, J_c, on the magnetic field H for the δ pinning for various pinning strengths is presented in Fig. 3(b). It vanishes as $(1-H/H_{c2})^{3/2}$ in the limit $H\rightarrow H_{c2}$. A similar dependence was obtained for the longitudinal (parallel to the magnetic field direction) critical current in Ref. 12.

The physical picture of the persistent current in the pinned state can be considered from two complementary angles. From one point of view, the nonzero persistent current appears in the pinned state due to shift and deformation of the vortex core, while from another point of view, the deformation is due to the pinning force creating the persistent current. Vortex cores are no longer circular, as was observed in numerical simulations by Priour and Fertig.[10] The deformation of the vortex cores is observable by currently existing scanning tunnel microscopy techniques.[13]

Most favorable conditions to look for the phenomena described in this paper are the following. Thermal fluctuations on the mesoscopic scale (not included in the calculation) ought to be minimized since they lead to thermal depinning of vortices at elevated temperatures. This means that for strongly fluctuating materials such as high T_c cuprates the temperature should be lower than the depinning temperature. Pinning should be strong enough similar to the one achieved in an array of artificial magnetic dots.

It is well known that a small deviation from the matching magnetic field leads to a sharp decrease of the critical current roughly to the level of an equivalent random pinning array.[4] Therefore there exists a sharp peak in the critical current of a small width ΔB. This is due to the fact that even for a small deviation from the matching condition, interstitial vortices (or vortex vacancies) appear and determine the reduced critical current. The current is still larger than that in the random pinning array with the same number of pinning sites because the interstitial vortices continue to move in the periodically modulated environment created by the pinned set of the vortex "channels."[14,15]

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